Time : 3 Hrs.
Marks : 360

JEE - MAIN : 2015

Date : 04-04-2015

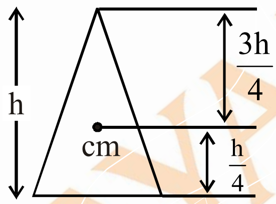
CODE - D

PHYSICS

Q.1 Distance of the centre of mass of a solid uniform cone from its vertex is z_0. If the radius of its base is R and its height is h then z_0 is equal to :

(1) $\frac{5h}{8}$ (2) $\frac{3h^2}{8R}$ (3) $\frac{h^2}{4R}$ (4) $\frac{3h}{4}$

Sol. (4)



Q.2 A red LED emits light at 0.1 watt uniformly around it. The amplitude of the electric field of the light at a distance of 1m from the diode is :

(1) 5.48 V/m (2) 7.75 V/m (3) 1.73 V/m (4) 2.45 V/m

Sol. (4)

$$I = \frac{E_0^2}{2c\mu_0}$$

$$\frac{P}{4\pi r^2} = \frac{E_0^2}{2c\mu_0}$$

$$E = \frac{P2c\mu_0}{4\pi r^2}$$

$$E = \frac{0.1 \times 2 \times 3 \times 10^{-8} \times 4\pi \times N^{-1}}{4\pi}$$

$$= \sqrt{6} = 2.45$$

Q.3 A pendulum made of a uniform wire of cross sectional area A has time period T. When an additional mass M is added to its bob, the time period change to T_M. If the Young's modulus of the material of the wire is Y then $\frac{1}{Y}$ is equal to :

(g = *gravitational acceleration*)

(1) $\left[1-\left(\frac{T_M}{T}\right)^2\right]\frac{A}{Mg}$ (2) $\left[1-\left(\frac{T}{T_M}\right)^2\right]\frac{A}{Mg}$ (3) $\left[\left(\frac{T_M}{T}\right)^2-1\right]\frac{A}{Mg}$ (4) $\left[\left(\frac{T_M}{T}\right)^2-1\right]\frac{Mg}{A}$

Sol. (3)

$$T = 2\pi\sqrt{\frac{\ell}{g}}$$

$$T_M = 2\pi\sqrt{\frac{\ell}{g}}$$

$$T_M = 2\pi\sqrt{\frac{\ell_0\left(1+\frac{F}{AY}\right)}{g}}$$

$$T_M = T_0\sqrt{\left(1+\frac{F}{AY}\right)}$$

$$\left(\frac{T_M}{T_0}\right)^2 = 1+\frac{F}{AY}$$

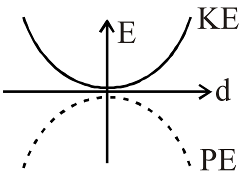
$$\frac{F}{AY} = \left(\frac{T_M}{T_0}\right)^2 - 1$$

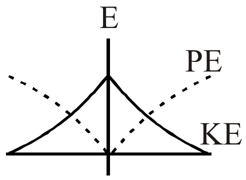
$$\frac{1}{Y} = \frac{A}{F}\left(1-\frac{T_M}{T_0}\right)^2$$

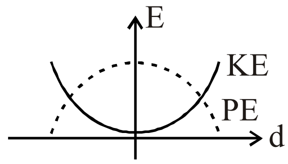
$$\frac{1}{Y} = \frac{A}{mg}\left(\left(\frac{T_M}{T_0}\right)^2 - 1\right)$$

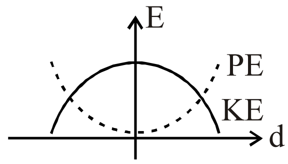
Q.4 For a simple pendulum, a graph is plotted between its kinetic energy (KE) and potential energy (PE) against its displacement d. Which one of the following represents these correctly ?

(graph are schematic and not drawn to scale)

(1) 

(2) 

(3) 

(4) 

Sol. (4)

$$V = \omega\sqrt{A^2 - d^2}$$

Kinetic energy $= \frac{1}{2}\omega^2 m\left(A^2 - d^2\right)$

$E = a - bd^2$ parabola

Q.5 A train is moving on a straight track with speed $20\,ms^{-1}$. It is blowing its whistle at the frequency of 1000 Hz. The percentage change in the frequency heard by a person standing near the track as the train passes him is (speed of sound $= 320\ ms^{-1}$) close to

(1) 18% (2) 24% (3) 6% (4) 12%

Sol. (4)

$$v' = v\left[\frac{V}{V \pm V_s}\right]$$

$$100\frac{\Delta v}{v} = 2\left({}^{V_s}\!/\!_{V}\right)(100)$$

$$= 200\left(\frac{20}{320}\right) = 12$$

Q.6 When 5V potential difference is applied across a wire of length 0.1 m, the drift speed of electrons $2.5\times10^{-4}\,ms^{-1}$. If the electron density in the wire is $8\times10^{28}\,m^{-3}$, the resistivity of the material is close to

(1) $1.6\times10^{-6}\,\Omega m$ (2) $1.6\times10^{-5}\,\Omega m$ (3) $1.6\times10^{-8}\,\Omega m$ (4) $1.6\times10^{-7}\,\Omega m$

Sol. (2)

$$\frac{\rho L}{A} = R = \frac{V}{i} = \frac{V}{neAV_d}$$

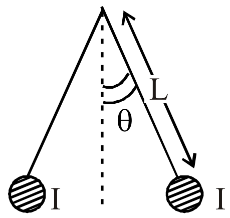
$$\rho = \frac{E}{L}\frac{}{neV_d} = \frac{\left(\frac{5}{0.1}\right)}{\left(8\times10^{28}\right)\left(1.6\times10^{-19}\right)\left(2.5\times10^{-4}\right)} = \left(\frac{50}{32}\right)10^{-5}$$

Q.7 Two long current carrying thin wires, both with current I, and held by insulating threads of length L and are in equilibrium as shown in the figure, with threads making an angle 'θ' with the vertical. If wires have mass λ per unit length then the value of I is :

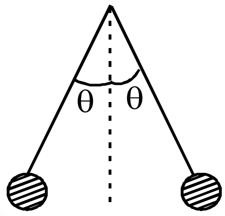
(g = gravitational acceleration)

(1) $2\sqrt{\frac{\pi gL}{\mu_0}\tan\theta}$ (2) $\sqrt{\frac{\pi\lambda gL}{\mu_0}\tan\theta}$

(3) $\sin\theta\sqrt{\frac{\pi\lambda gL}{\mu_0\cos\theta}}$ (4) $2\sin\theta\sqrt{\frac{\pi\lambda gL}{\mu_0\cos\theta}}$



Sol. (4)

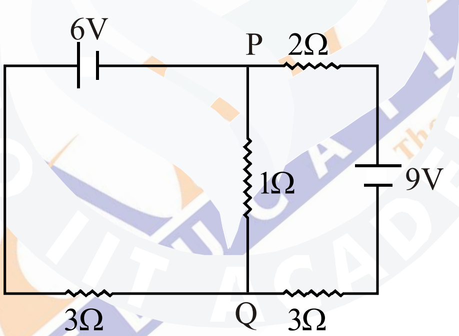


$$T\sin\theta = \frac{\mu_0 I^2 x}{4\pi L\sin\theta}$$

$$T\cos\theta = \lambda xg$$

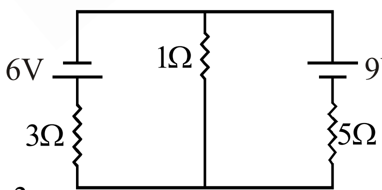
$$\text{Tan}\,\theta = \frac{\mu_0 I^2}{4\pi L\sin\lambda g} \Rightarrow I = 2\sin\theta\sqrt{\frac{\pi\lambda gL}{\mu_0\cos\theta}}$$

Q.8 In the circuit shown, the current in the 1Ω resistor is :



(1) 0.13 A, from Q to P (2) 0.13 A, from P to Q (3) 1.3 A, from P to Q (4) 0A

Sol. (1)



$$E_{eq} = \frac{\frac{5}{5}\frac{6}{3} - \frac{9}{5}\times\frac{3}{3}}{\frac{1}{3}+\frac{1}{5}} = \frac{3}{\cancel{15}\times\frac{8}{\cancel{15}}} = \frac{3}{8}$$

$$r_{eq} = \frac{3\times5}{8} = \frac{15}{8}$$

$$\boxed{i = \frac{\frac{3}{8}}{\frac{15}{8}+1} = \frac{3}{\cancel{8}\times\frac{23}{\cancel{8}}} = \frac{3}{23}}$$

= 0.13

Q.9 Assuming human pupil to have a radius of 0.25 cm and a comfortable viewing distance of 25 cm, the minimum separation between two objects that human eye can resolve at 500 nm wavelength is

(1) $100\,\mu m$ (2) $300\,\mu m$ (3) $1\,\mu m$ (4) $30\,\mu m$

Sol. (4)

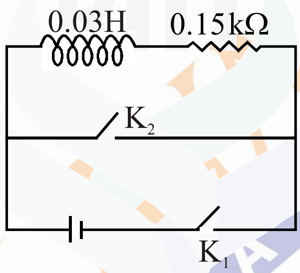
$$2\theta = \frac{1.22\lambda}{d}$$

$$2\theta = 1.22 \times 5 \times 10^{-6}$$

$$d = R\,2\theta = \frac{1}{4} \times 1.22 \times 5 \times 10^{-6}$$

$$\cong 30\,\mu m$$

Q.10 An inductor (L = 0.03 H) and a resistor $(R = 0.15\,k\Omega)$ are connected in series to a battery of 15V EMF in a circuit shown below. The key K_1 has been kept closed for a long time. Then at $t = 0, K_1$ is opened and key K_2 is closed simultaneously. At t = 1ms, the current in the circuit will be : $(e^5 \cong 150)$



(1) 6.7 mA (2) 0.67 mA (3) 100 mA (4) 67 mA

Sol. (2)

$$i = \frac{15}{0.15 \times 10^{+3}}$$

$$15 \times 10^{-2} \times 10^{+3}$$

$$1 = 10^{-1} \quad e^{-1\frac{Rt}{L}}$$

$$= \frac{1}{150} \qquad \frac{0.15 \times 10^{3}}{0.03} \times 10^{-3}$$

$$= \frac{1}{1500}$$

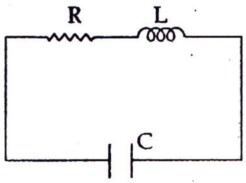
$$= 1 \qquad 10^{-2}$$

$$= 6.67 \times 10^{-4}$$

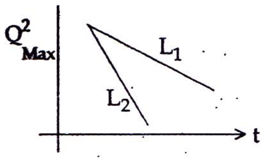
$$= 0.667 \times 10^{-3}$$

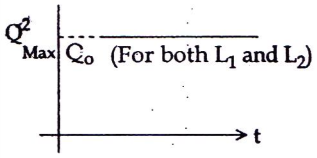
$$= 0.667\,mA$$

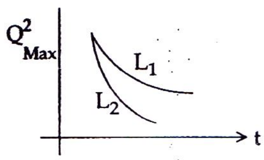
Q.11 An LCR circuit is equivalent to a damped pendulum. In an LCR circuit the capacitor is charged to Q_0 and then connected to the L and R as shown below :

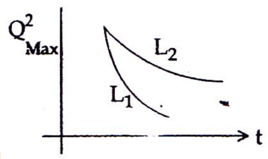


If a student plots graphs of the square of maximum charge $\left(Q^2_{Max}\right)$ on the capacitor with time (t) for two different values L_1 and L_2 $(L_1 > L_2)$ of L then which of the following represents this graph correctly ? *(plots are schematic and not drawn ot scale)*

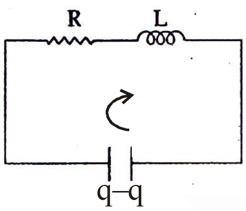
(1) 

(2) 

(3) 

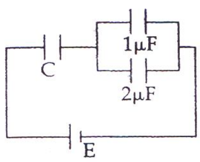
(4) 

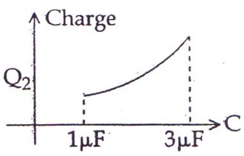
Sol. **(3)**

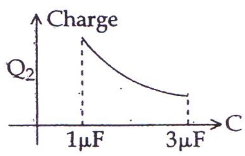


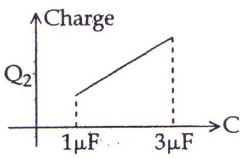
$$-\frac{q}{C} - iR - \frac{Ldi}{dT} = 0$$

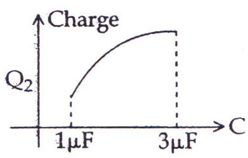
Q.12 In the given circuit, charge Q_2 on the $2\mu F$ capacitor changes as C is varied from $1\mu F$ to $3\mu F$. Q_2 as a function of 'C' is given properly by : (figures are drawn schematically and are not to scale)



(1) 

(2) 

(3) 

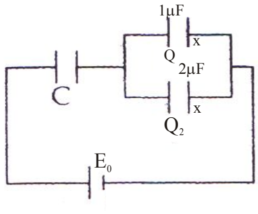
(4) 

Sol. **(4)**

$$C_{eq} = \frac{3C}{C+3}$$

$$Q = \frac{3C\,E_0}{C+3}$$

$$Q_2 = \frac{2}{3}\left(\frac{3CE_0}{C+3}\right) = \frac{2CE_0}{C+3}$$



$$C = 1 \quad \Rightarrow \quad Q_2 = \frac{2E_0}{4} = \frac{E_0}{2}$$

$$C = 3 \quad \Rightarrow \quad Q_2 = \frac{2\times 3\times E_0}{2\times 3} = E_0$$

$$Q_2 = 2E_0\left[\frac{C}{C+3}\right]$$

$$\frac{dQ_2}{dC} = 2E_0\left[\frac{(C+3)\times 1-(C)\times 1}{(C+3)^2}\right] = 2E_0\frac{(C+3-C)}{(C+3)^2}$$

$$dQ_2 = \frac{6E_0}{(C+3)^2} \quad \text{slope decreases}$$

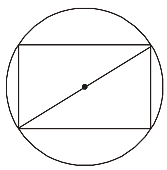
Q.13 From a solid sphere of mass M and radius R a cube of maximum possible volume is cut. Moment of inertia of cube about an axis passing through its center and perpendicular to one of its faces is :

(1) $\frac{4MR^2}{9\sqrt{3}\pi}$ (2) $\frac{4MR^2}{3\sqrt{3}\pi}$ (3) $\frac{MR^2}{32\sqrt{2}\pi}$ (4) $\frac{MR^2}{16\sqrt{2}\pi}$

Sol. **(1)**

$$2R = \sqrt{3}a$$

$$\frac{2R}{\sqrt{3}} = a$$



$$I = \frac{\text{mass}(\text{side})^2}{6}$$

$$= \left(\frac{M}{\frac{4}{3}\pi R^3}\right)a^3\,\frac{4R^2}{3}\,\frac{1}{6}$$

$$= \frac{3M}{4\pi R^3}\,\frac{8R^3}{3\sqrt{3}}\,\frac{4R^2}{3\times 6}$$

$$= \frac{8MR^2}{18\sqrt{3}\pi} = \frac{4MR^2}{\pi 9\sqrt{3}}$$

Q.14 The period of oscillation of a simple pendulum is $T = 2\pi\sqrt{\frac{L}{g}}$. Measured value of L is 20.0 cm known to 1 mm accuracy and time for 100 oscillations of the pendulum is found to be 90 s using a wrist watch of 1 s resolution. The accuracy in the determination of g is :

(1) 1% (2) 5% (3) 2% (4) 3%

Sol. **(4)**

$$T = 2\pi\sqrt{\frac{L}{g}}$$

$$\frac{t}{n} = 2\pi\sqrt{\frac{L}{g}}$$

$$\frac{t^2}{n^2} = 4\pi^2\frac{L}{g}$$

$$g = \frac{4\pi L^2 n^2}{t^2}$$

$$\ln g = \ln 4\pi n^2 + 2\ln L - 2\ln t$$

$$\frac{dg}{g} = \frac{2\Delta L}{L} - 2\frac{dt}{t}$$

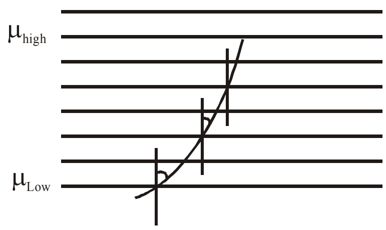
$$\frac{dg}{g}\times 100 = 2\times\frac{0.1}{20}\times 100 + 2\frac{1}{90}\times 100 = 1 + \frac{20}{9} \simeq 3\%$$

Q.15 On a hot summer night, the refractive index of air is smallest near the ground and increases with height from the ground. When a light beam is directed horizontally, the Huygens' principle leads us to conclude that as it travels, the light beam :

(1) bends downwards (2) bends upwards

(3) becomes narrower (4) goes horizontally without any deflection

Sol. **(2)**



As we move up "θ" decreases bends upwards

Q.16 A signal of 5 kHz frequency is amplitude modulated on a carrier wave of frequency 2 MHz. The frequencies of the resultant signal is / are :

(1) 2005 kHz, 2000 kHz and 1995 kHz (2) 2000 kHz and 1995 kHz

(3) 2 MHz only (4) 2005 kHz, and 1995 kHz

Sol. **(1)**

$A_M - A_C$	A_M	$A_M + A_C$
1995 KHz	2000	2005

Q.17 A solid body of constant heat capacity 1 J/°C is being heated by keeping it in contact with reservoirs in two ways :

(i) Sequentially keeping in contact with 2 reservoirs such that each reservoir supplies same amount of heat.

(ii) Sequentially keeping in contact with 8 reservoirs such that each reservoir supplies same amount of heat.

In both the cases body is brought from initial temperature 100°C to final temperature 200°C. Entropy change of the body in the two cases respectively is :

(1) $ln2, 2ln2$ (2) $2ln2, 8ln2$ (3) $ln2, 4ln2$ (4) $ln2, ln2$

Sol. **(4)**

$$\Delta s = \int \frac{dQ}{T} = \int_{100}^{200} mc\frac{dT}{T} \qquad = mc\int_{100}^{200} \frac{dT}{T}$$

or $\Delta s = mc \ln 2$

$$= \ln 2 \left(\because mc = 1\right)$$

Q.18 Consider a spherical shell of radius R at temperature T. The black body radiation inside it can be considered as an ideal gas of photons with internal energy per unit volume $u = \frac{U}{V} \propto T^4$ and pressure $p = \frac{1}{3}\left(\frac{U}{V}\right)$. If the shell now undergoes an adiabatic expansion the relation between T and R is :

(1) $T \propto \frac{1}{R}$ (2) $T \propto \frac{1}{R^3}$ (3) $T \propto e^{-R}$ (4) $T \propto e^{-3R}$

Sol. **(1)**

From given Informaiton $P \propto u$

$$P \propto T^4$$

$$\frac{T}{V} \propto T^4 \qquad \frac{1}{V} \propto T^3$$

$$T^3V = \text{costant}$$

$$T^3R^3 = \text{costant}$$

$$TR = \text{costant}$$

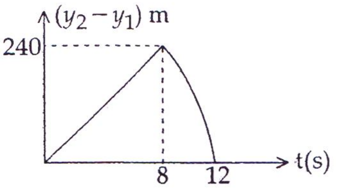
$$\boxed{T \propto \frac{1}{R}}$$

Q.19 Two stones are thrown up simultaneously from the edge of a cliff 240 m high with initial speed of 10 m/s and 40 m/s respectively. Which of the following graph best represents the time variation of relative position of the second stone with respect of the first ?

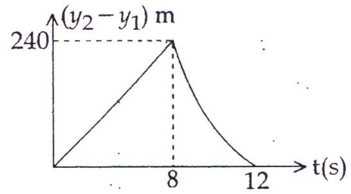
(Assume stones do not rebound after hitting the ground and neglect air resistance, take $g = 10\ m/s^2$)

(The figure are schematic and not drawn to scale)

(1)



(2)



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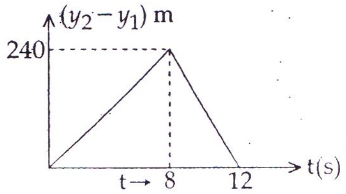

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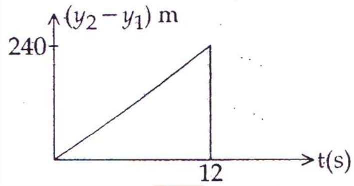


Sol. **(1)**

The displacement equation of the first object is

$$y_1 = 10t - \frac{1}{2}gt^2 = 10t - 5t^2$$

and the final displacement is $y_1 = -240m$

so, $-240 = 10t - 5t^2$

$$5t^2 - 10t - 240 = 0$$

$$t^2 - 2t - 48 = 0$$

$$t = \frac{2 \pm \sqrt{2^2 + 4 \times 48}}{2} = 8\,sec$$

time taken to reach the ground is 8sec,

In the same way second object's displacement equation is $y_2 = 40t - 5t^2$

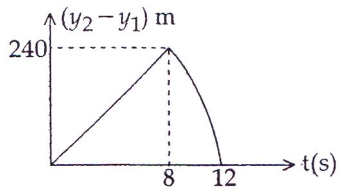
and the taken to reach the ground is $t = 12\,sec$.

The relative displacement is

$$y_2 - y_1 = 30t \qquad \text{when}\,(0 < t \le 8\,sec)$$

$$y_2 - y_1 = 40t - 5t^2 \qquad \text{when}\,(8 \le t \le 12\,sec)$$

So graph is



Q.20 A uniformly charged solid sphere of radius R has potential V_0 (measured with respect to ∞) on its surface. For this sphere the equipotential surfaces with potentials $\frac{3V_0}{2}, \frac{5V_0}{4}, \frac{3V_0}{4}$ and $\frac{V_0}{4}$ have radius R_1, R_2, R_3 and R_4 respectively. Then

(1) $R_1 = 0$ and $R_2 < (R_4 - R_3)$

(2) $2R < R_4$

(3) $R_1 = 0$ and $R_2 > (R_4 - R_3)$

(4) $R_1 \neq 0$ and $(R_2 - R_1) > (R_4 - R_3)$

Sol. **(1)**

Assuming sphere carries positive charge,

Potential on the surface is $V_0 = \frac{KQ}{R}$

Potential expression inside the sphere is $V_{inside} = \frac{KQ}{2R^3}(3R^2 - r^2)$

Potential at the centre is $V_{centre} = \frac{3KQ}{2R}(\because r = 0) = \frac{3V_0}{2}$

So radius of equi-potential at the centre of sphere is zero. $R_1 = 0$

Let radius of 2nd equi - potential surface is R_2 then

$$\frac{5V_0}{4} = \frac{KQ}{2R^3}(3R^2 - R_2^2)$$

$$\frac{5V_0}{1/2} = \frac{V_0}{2R^3}(3R^2 - R_2^2)$$

$$5R^2 = 6R^2 - 2R_2^2$$

$$2R_2^2 = R^2 \Rightarrow R_2 = \frac{R}{\sqrt{2}}$$

Let radius of 3rd equi-potential is R_3 then

$$\frac{3V_0}{4} = \frac{KQ}{R_3}$$

$$\frac{3KQ}{4R} = \frac{KQ}{R_3} \Rightarrow R_3 = \frac{4R}{3}$$

Let radius of 4th equi-potential is R_4 then

$$\frac{V_0}{4} = \frac{KQ}{R_4}$$

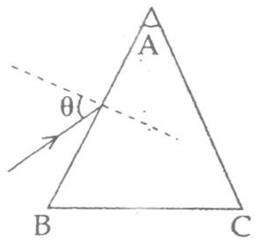
$$\frac{KQ}{4R} = \frac{KQ}{R_4} \Rightarrow R_4 = 4R$$

$$R_4 - R_3 = 4R - \frac{4R}{3} = \frac{8R}{3} = 2.66R$$

$$R_2 = \frac{R}{1.414} < R$$

So, $R_2 < R_4 - R_3$

Q.21 Monochromatic light is incident on a glass prism of angle A. If the refractive index of the material of the prism is μ, a ray, incident at an angle θ, on the face AB would get transmitted through the face AC of the prism provided.



(1) $\theta > \cos^{-1}\left[\mu \sin\left(A + \sin^{-1}\left(\frac{1}{\mu}\right)\right)\right]$

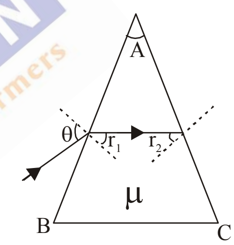
(2) $\theta < \cos^{-1}\left[\mu \sin\left(A + \sin^{-1}\left(\frac{1}{\mu}\right)\right)\right]$

(3) $\theta > \sin^{-1}\left[\mu \sin\left(A - \sin^{-1}\left(\frac{1}{\mu}\right)\right)\right]$

(4) $\theta < \sin^{-1}\left[\mu \sin\left(A - \sin^{-1}\left(\frac{1}{\mu}\right)\right)\right]$

Sol. (3)

By snell's law at 1st refracting surface is $1.\sin\theta = \mu.\sin r_1$



$$\sin r_1 = \frac{\sin\theta}{\mu}$$

$r_2 < \theta_1$, ray to transmitted through surface AC,

$$r_2 < \theta_c$$

$$r_2 < \sin^{-1}\left(\frac{1}{\mu}\right)$$

$$A - r_1 < \sin^{-1}\left(\frac{1}{\mu}\right)$$

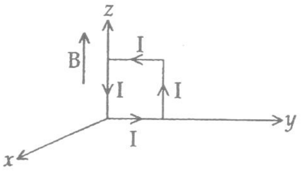
$$A - \sin^{-1}\left(\frac{\sin\theta}{\mu}\right) < \sin^{-1}\left(\frac{1}{\mu}\right)$$

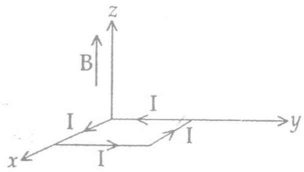
$$A - \sin^{-1}\left(\frac{1}{\mu}\right) < \sin^{-1}\left(\frac{\sin\theta}{\mu}\right)$$

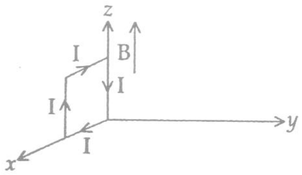
$$\sin\left(A - \sin^{-1}\left(\frac{1}{\mu}\right)\right) < \frac{\sin\theta}{\mu} \qquad \mu\sin\left(A - \sin^{-1}\left(\frac{1}{\mu}\right)\right) < \sin\theta$$

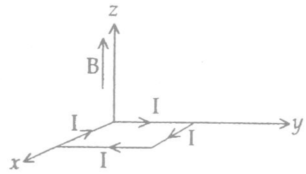
$$\theta > \sin^{-1}\left(\mu\sin\left(A - \sin^{-1}\left(\frac{1}{\mu}\right)\right)\right)$$

Q.22 A rectangular loop of sides 10 cm and 5 cm carrying a current I of 12A is placed in different orientations as shown in the figures below :

(a) 

(b) 

(c) 

(d) 

If there is a uniform magnetic field of 0.3 T in the positive z direction, in which orientations the loop would be in (i) stable equilibrium and (ii) unstable eqilibrium ?

(1) (b) and (d), respectively
(2) (b) and (c), respectively
(3) (a) and (b) , respectively
(4) (a) and (c), respectively

Sol. (1)

is in stable equilibrium because $i\vec{A}$ *and* $\vec{B}$

are in same direction is in unstable equilibrium $i\vec{A}$ *and* $\vec{B}$ are opposite in direction

Q.23 Two coaxial solenoids of different radii carry current I in the the inner soleoid due to the outer one and $\vec{F}_2$ be the magnetic force on the outer solenoid due to the inner one. Then :

(1) $\vec{F}_1$ is radially inwards and $\vec{F}_2 = 0$
(2) $\vec{F}_1$ is radially outwards and $\vec{F}_2 = 0$
(3) $\vec{F}_1 = \vec{F}_2 = 0$
(4) $\vec{F}_1$ is radially inwards and $\vec{F}_2$ is radially onwards

Sol. (3)

Outer solenoid is not in the magnetic field of inner solenoid.Hence magnetic force on outer solenoid due to inner solenoid. Hence no force on inner solenoid due to outer solenoid.

Q.24 A particle of mass m moving in the x direction with speed $2v$ is hit by another particle of mass $2m$ moving in the y direction with speed v. If the collision is perfectely inelastic, the percentage loss in the energy during the collision is close to :

(1) 56 %
(2) 62 %
(3) 44 %
(4) 50 %

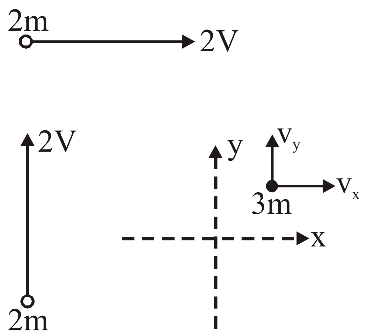
Sol. (1)

along x-axis

$$2m(2v) = 3m.v_x \Rightarrow v_x = \frac{2v}{3}$$

$$2m(v) = 3mv_y \Rightarrow v_y = \frac{2v}{3}$$

$$v_{net} = \sqrt{v_x^v + u_y^v} = \frac{2\sqrt{2}v}{3}$$



$$= \frac{1}{2}m(2v)^2 + \frac{1}{2}(2m)v^2 - \frac{1}{2}(3m)\left(\frac{2\sqrt{2}}{3}\right)^2$$

$$= 3mv^2 - \frac{4mv^2}{3} = \frac{5mv^2}{3}$$

$$= \frac{loss\ in\ K.E}{Initial\ K.E} \times 100$$

$$= \frac{5/3mv^2}{3mv^2} \times 100 = 5/9 \times 100 = 56\%$$

Q.25 Consider an ideal gas confined in an isolated closed chamber. As the gas undergoes an adiabatic expansion, the average time of collision between molcules increases as V^q, where V is the volume of the gas. The value of q is : $\left(\gamma = \frac{C_p}{C_v}\right)$

Sol. (1)

R.M.S velocity of molecule is $V_{rms} = \sqrt{\frac{3RT}{M}}$

$$V_{rms} \propto \sqrt{T}$$

and time of collision is t $t = \frac{d}{V_{rms}}$

$$t \propto \frac{1}{\sqrt{t}}$$

$$t \propto T^{1/2}$$

In adiabatic expansion TV^{r-1} = constant

$$T \propto V^{1-r}$$

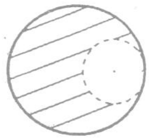
so, $t \propto T^{-1/2}$

$$t \propto \left(V^{1+r}\right)^{-1/2}$$

$$t \propto V^{\frac{r+1}{2}}$$

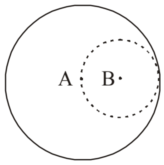
So, $q = \frac{r+1}{2}$

Q.26 From a solid sphere of mass M and radius R, a spherical portion of radius $\frac{R}{2}$ is removed, as shown in the figure. Taking gravitional potential $V = 0$ at $r = \infty$, the potential at the centre of cavity thus formed is:
(G = gravitational constant)



(1) $\frac{-2GM}{3R}$ (2) $\frac{-2GM}{R}$ (3) $\frac{-GM}{2R}$ (4) $\frac{-GM}{R}$

Sol. (4)

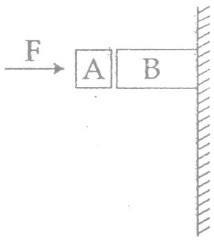


$V_B = V_A + V_B$

$V_B = V_M - V_m$

$= \frac{-11GM}{8R} + \frac{3GM}{8R} = \frac{-GM}{R}$

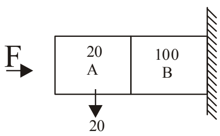
Q.27



Given in the figure are two blocks A and B of weight 20 N and 100N respectively. There are being pressed against a wall by a force F as shown. If the coefficient of friction between the blocks is 0.1 and between block B and the wall is 0.15, the frictional force applied by the wall on block B is :

(1) 120 N (2) 150 N (3) 100 N (4) 80 N

Sol. (1)



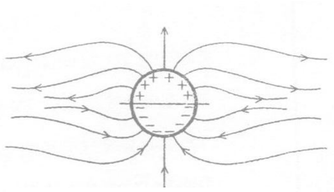
F = 120

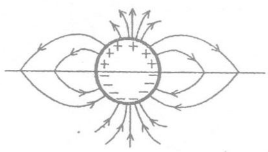
F = 120

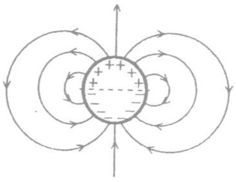
If the system is not accelertion

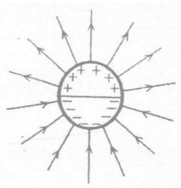
Q.28 A long cylinderal shell carries positive surface charge σ in the upper half and negative surface charge $-\sigma$ in the lower half. The electric field lines around the cylinder will looke like figure given in :

(*Figures are schematic and not drawn to scale*)

(1) 

(2) 

(3) 

(4) 

Sol. (3)

Lines emerge out from positive and terminate in to negative charge.

Q.29 As an electron makes a transition from an excited state to the ground state of a hydrogen - like atom / ion:

(1) kinetic energy decrease, potential energy increase but total energy remians same

(2) kinetic energy and total energy decrease but potential energy increases

(3) its kinetic energy increases but potential energy and total energy decrease

Sol. (3)

Speed of electron in the orbit is given by

$$V_n \propto \frac{z}{n}$$

An orbit number decreases, velocity of electron increases and results in increase in K.E.

Potential energy is given by P.E. = – 2K.E, so Potential energy decreases

Total energy is given by T.E. = – K.E., so total energy of electron also decreases.

Q. 30 Match List - I (Fundamental Experiment)

with List - II (its conclusion) and select the correct option from the choices given below the list :

List - I	**List - II**
(A) Franck-Hertz Experiment	(i) Particle nature of light
(B) Photo-electric experiment	(ii) Discrete energy levels of atom
(C) Davison - Germer Experiment	(iii) Wave nature of electron
	(iv) Structure of atom

(1) (A) - (ii) (B) - (i) (C) - (iii)
(2) (A) - (iv) (B) - (iii) (C) - (ii)
(3) (A) - (i) (B) - (iv) (C) - (iii)
(4) (A) - (ii) (B) - (iv) (C) - (iii)

Ans: (1)

Sol: Photelectric effect expriment explains the Particle nature of light

Davison - Germer Experiment explains Wave nature of electron

Franck-Hertz Experiment explains Discrete energy levels of atom

MATHEMATICS

Q.31 Let $\vec{a}, \vec{b}$ and $\vec{c}$ be three non - zero vectors such that no two of them are collinear and $(\vec{a}\times\vec{b})\times\vec{c} = \frac{1}{3}|\vec{b}||\vec{c}|\vec{a}$. If θ is the angle between vectors $\vec{b}$ and $\vec{c}$, then a value of $\sin\theta$ is :

(1) $\frac{2}{3}$ (2) $\frac{-2\sqrt{3}}{3}$ (3) $\frac{2\sqrt{2}}{3}$ (4) $\frac{-\sqrt{2}}{3}$

Sol: (3)

$$(\vec{a}\times\vec{b})\times\vec{c} = \frac{1}{3}|\vec{b}||\vec{c}|\vec{a}$$

$$\Rightarrow (\vec{a}.\vec{c})\vec{b} - (\vec{b}.\vec{c})\vec{a} = -\frac{1}{3}|\vec{b}||\vec{c}|$$

$$\Rightarrow \vec{b}.\vec{c} = -\frac{1}{3}|\vec{b}||\vec{c}|$$

$$\Rightarrow \frac{\vec{b}.\vec{c}}{|\vec{b}||\vec{c}|} = -\frac{1}{3}$$

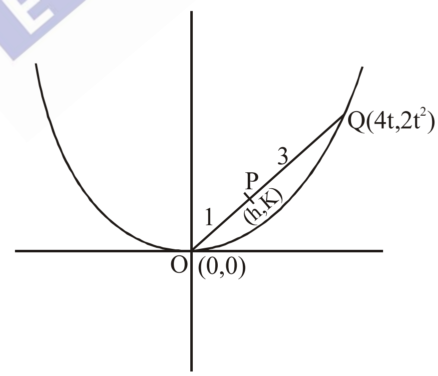
$$\Rightarrow \cos\theta = -\frac{1}{3}$$

Hence $\sin\theta = \frac{2\sqrt{2}}{3}$

Q.32 Let O be the vertex and Q be any point on the parabola, $x^2 = 8y$. If the point P divides the line segment OQ internally in the ratio 1:3, then locus of P is :

(1) $y^2 = 2x$ (2) $x^2 = 2y$ (3) $x^2 = y$ (4) $y^2 = x$

Sol: (2)



Any point on the curve $x^2 = 8y$
is $(4t, 2t^2)$
Point P(h,x) divides the line segment joining OQ in ratio 1 : 3

$$\Rightarrow h = \frac{4t}{4} = t \;\&\; k = \frac{2t^2}{4} = \frac{t^2}{2}$$

Hence locus of point P is $x^2 = 2y$

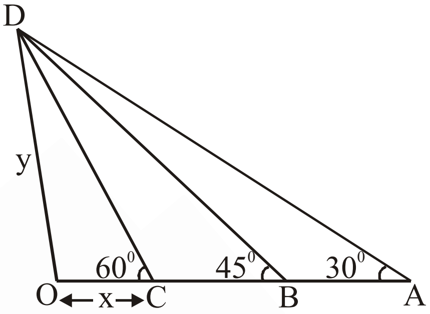
Q.33 If the angles of elevation of the top of a tower from three collinear points A,B and C, on a line leading to the foot of the tower, are 30^0, 45^0 and 60^0 respectively, then the ratio, AB : BC, is :

(1) $1:\sqrt{3}$ (2) 2 : 3 (3) $\sqrt{3}:1$ (4) $\sqrt{3}:\sqrt{2}$

Sol: (3)

from figure $\tan 60^0 = \dfrac{y}{x} \Rightarrow y = \sqrt{3}\,x........(1)$

$\tan 45^0 = \dfrac{y}{x+BC}$



$\Rightarrow x + BC = y = \sqrt{3}x$

$\Rightarrow BC = \left(\sqrt{3}-1\right)x..........(ii)$

$\tan 30^0 = \dfrac{y}{x+CA}$

$\Rightarrow x + CA = \sqrt{3}\,y = 3x$

$\Rightarrow CA = 2x$

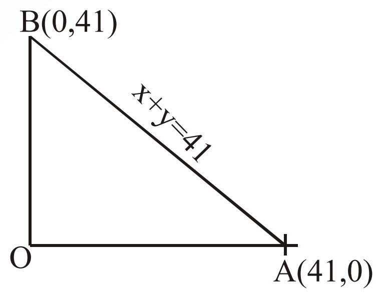
$\Rightarrow AB = CA - BC = 2x - \left(\sqrt{3}-1\right)x = \sqrt{3}\left(\sqrt{3}-1\right)x........(iii)$

Hence $AB:BC = \sqrt{3}\left(\sqrt{3}-1\right)x:\left(\sqrt{3}-1\right)x = \sqrt{3}:1$

Q.34 The number of points, having both co-ordinates as integers, that lie in the interior of the triangle with vertices $(0,0),(0,41)$ and $(41,0)$, is

(1) 820 (2) 780 (3) 901 (4) 861

Sol: (2)



Now number of integral points lie in side the triangle are
1) If x = 1, then y may be 1,2,3......39
2) If x = 2, then y may be 1,2,3......38
and so on
If x = 39 , then value of y is 1 only. Hence number of interior points are

$1 + 2 + 3 + ... + 39 = \dfrac{39\times 40}{2} = 780$

Q.35 The equation of the plane containing the line $2x-5y+z=3;\ x+y+4z=5$, and parallel to the plane, $x+3y+6z=1$, is :

(1) $x+3y+6z=7$ (2) $2x+6y+12z=-13$

(3) $2x+6y+12z=13$ (4) $x+3y+6z=-7$

Sol: (1)

Equation of the plane containing the line

$2x-5y+7-3=0=x+y+47-5$

is $\left(2x-5y+7-3\right)+\lambda\left(x+y+47-7\right)=0$

$\Rightarrow\left(2+\lambda\right)x+\left(\lambda-5\right)y+\left(4\lambda+1\right)7-\left(3+5\lambda\right)=0$ ________(1)

As equation (1) is parallel to the plane $x+3y+6z=1$

then $\frac{2+\lambda}{1}=\frac{\lambda-5}{3}=\frac{4\lambda+1}{6}$ ....(2)

from (2) $\lambda=\frac{-11}{2}$

using $\lambda=\frac{-11}{2}$ in equation (1) we have

equation of plane is $x+3y+6z=7$

Q.36 Let A and B be two sets containing four and two elements respectively. Then the number of subsets of the set $A\times B$, each having at least three elements is :

(1) 275 (2) 510 (3) 219 (4) 256

Sol: (3)

$n(A)=4\,and\,n(B)=2$

$\Rightarrow \eta(A\times B)=8$

Hence number of subsets of $(A\times B)$
each having at least three eleements is equal to

${}^{8}C_{3}+{}^{8}C_{4}+{}^{8}C_{5}+......+{}^{8}C_{8}$

$=2^{8}-{}^{8}C_{0}-{}^{8}C_{1}-{}^{8}C_{2}$

$=256-1-8-28=219$

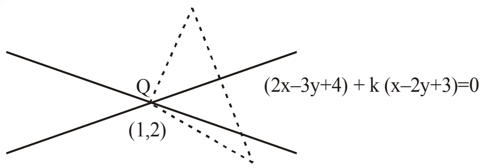
Q.37 Locus of the image of the point $(2,3)$ in the line $(2x-3y+4)+k(x-2y+3)=0,\ k\in R,$ is a :

(1) circle of radius $\sqrt{2}$ (2) circle of radius $\sqrt{3}$

(3) straight line parallel to x - axis (4) straight line parallel to y - axis

Sol: (1)

The given variable lines is the family lines intersecting at (1,2)
The mirror image P' of the given point P and the point P
is equidistant from the point (1,2), hence locus of P'
is a circle with centre (1,2) and radius $PQ=\sqrt{2}$



Q.38 $\lim\limits_{x\to 0}\frac{(1-\cos 2x)(3+\cos x)}{x\tan 4x}$ is equal to :

(1) 2 (2) $\frac{1}{2}$ (3) 4 (4) 3

Sol: (1)

$$\lim_{x\to 0}\frac{(1-\cos 2x)(3+\cos x)}{x\tan 4x}$$

$$=\lim_{x\to 0}\frac{2\sin^{2}x(3+\cos x)}{x^{2}\frac{\tan 4x}{4x}\cdot 4}$$

$$=2\lim_{x\to 0}\frac{\sin^{2}x}{x^{2}}\cdot\lim_{x\to 0}(3+\cos x)\cdot\lim_{x\to 0}\frac{1}{\frac{\tan 4x}{4x}}\cdot\frac{1}{4} \qquad =2:1.4\cdot\frac{1}{4}=2$$

Q.39 The distance of the point $(1,0,2)$ from the point of intersection of the line $\frac{x-2}{3}=\frac{y+1}{4}=\frac{z-2}{12}$ and the plane $x-y+z=16$, is :

(1) $3\sqrt{21}$ (2) 13 (3) $2\sqrt{14}$ (4) 8

Sol: (2)

Parametric co-ordinate of any point on the line $\frac{x-2}{3}=\frac{y+1}{4}=\frac{z-2}{12}$ is

$x=3\lambda+2, y=4\lambda-1, z=12\lambda+2$

If this point lies on the plane $x-y+x=15$

then $(3\lambda+2)-(4\lambda-1)+(12\lambda+2)=16$

$\Rightarrow 11\lambda=11 \Rightarrow \lambda=1$

$\Rightarrow$ The point at intersection = (5, 3, 14)

$\Rightarrow$ distance = 13.

Q.40 The sum of coefficients of integral powers of x in the binomial expansion of $\left(1-2\sqrt{x}\right)^{50}$ is

(1) $\frac{1}{2}\left(3^{50}-1\right)$ (2) $\frac{1}{2}\left(2^{50}+1\right)$ (3) $\frac{1}{2}\left(3^{50}+1\right)$ (4) $\frac{1}{2}\left(3^{50}\right)$

Sol: (3)

$$\left(1-2\sqrt{x}\right)^{50}={}^{50}C_0-{}^{50}C_1\left(2\sqrt{x}\right)+{}^{50}C_2\left(2\sqrt{x}\right)^2-{}^{50}C_3\left(2\sqrt{x}\right)^3+.....$$

$$\left(1+2\sqrt{x}\right)^{50}={}^{50}C_0+{}^{50}C_1\left(2\sqrt{x}\right)+{}^{50}C_2\left(2\sqrt{x}\right)^2+{}^{50}C_3\left(2\sqrt{x}\right)^3+.....$$

$$\left(1-2\sqrt{x}\right)^{50}+\left(1+2\sqrt{x}\right)^{50}=2\left[{}^{50}C_0+{}^{50}C_2 2^2x+{}^{50}C_4 2^3x^2+.....\right]$$

The required sum is obtained by putting x = 1 above as $\frac{1+3^{50}}{2}$.

Q.41 The sum of first 9 terms of the series $\frac{1^3}{1}+\frac{1^3+2^3}{1+3}+\frac{1^3+2^3+3^3}{1+3+5}+.....$ is :

(1) 142 (2) 192 (3) 71 (4) 96

Sol: (4)

$$T_r=\frac{1^3+2^3+......+r^3}{1+3+5+.....+(2r-1)}=\frac{r^2(r+1)^2}{4(r^2)} \qquad =\frac{(r+1)^2}{4}$$

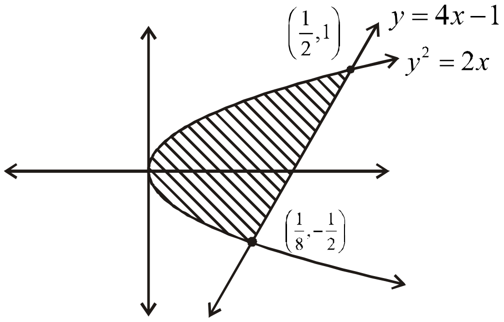
$$\Rightarrow sum=\sum_{r=1}^{9}T_r=\sum_{r=1}^{n}\frac{(r+1)^2}{4}$$

$$=\frac{1}{4}\left[2^2+3^2+......+10^2\right]=\frac{1}{4}\left[1^2+2^2+......+10^2-1\right] \qquad =96$$

Q.42 The area (in sq. units) of the region described by

(1) $\frac{15}{64}$ (2) $\frac{9}{32}$ (3) $\frac{7}{32}$ (4) $\frac{5}{64}$

Sol: (2)



The required region is shown shaded. The points of intersections are $\left(\frac{1}{8},-\frac{1}{2}\right)$ & $\left(\frac{1}{2},1\right)$

$$\Rightarrow \text{Required area} = \int_{-\frac{1}{2}}^{1}\left(\frac{y+1}{4}-\frac{y^2}{2}\right)dy = \frac{9}{32}$$

Q.43 The set of all values of λ for which the system of linear equations:

$2x_1 - 2x_2 + x_3 = \lambda x_1$

$2x_1 - 3x_2 + 2x_3 = \lambda x_2$

$-x_1 + 2x_2 \quad = \lambda x_3$

has a non- trivial solution,

(1) Contains two elements

(2) Contains more than two elements.

(3) Is an empty set

(4) Is a singleton.

Sol: (1)

To have Non- Trivial solution

$$D=0 \Rightarrow \begin{vmatrix} \lambda-2 & 2 & -1 \\ 2 & -3-\lambda & 2 \\ -1 & 2 & -\lambda \end{vmatrix} = 0$$

$\Rightarrow \lambda^3+\lambda^2-5\lambda+3=0$

$\lambda = 1,1,3$

Hence λ have 2 Values

Q.44 A complex number z is said to be unimodular if $|z|=1$. Suppose z_1 and z_2 are complex numbers such that $\frac{z_1-2z_2}{2-z_1\bar{z}_2}$ is unimodular and z_2 is not unimodular. Then the point z_1 lies on a :

(1) Circle of radius 2

(2) Circle of radius $\sqrt{2}$

(3) Straight line parallel to x-axis

(4) Straight line parallel to y-axis

Sol: (1)

$$\left|\frac{z_1-2z_2}{2-z_1\bar{z}_2}\right|^2=1$$

$$\Rightarrow\left(\frac{z_1-2z_2}{2-z_1\bar{z}_2}\right)\left(\frac{\bar{z}_1-2\bar{z}_2}{2-\bar{z}_1-z_2}\right)=1$$

$$\Rightarrow |z_1|^2|z_2|^2-|z_1|^2-4|z_2|^2+4=0$$

$$\Rightarrow\left(|z_1|^2-1\right)\left(|z_1|^2-4\right)=0$$

$$\Rightarrow|z_1|=2$$

Circle with radius “2”

Q.45 The number of common tangents to the circles $x^2+y^2-4x-6y-12=0$ and $x^2+y^2+6x+18y+26=0,$ is

(1) 3 (2) 4 (3) 1 (4) 2

Sol: (1)

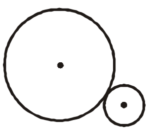
$C_1=(2,3)\quad ,r_1=5$

$C_2=(-3,-9)\ \ ,r_2=8$

$C_1C_2=13\quad ,\quad r_1+r_2=13$

$C_1C_2=\ r_1+r_2$

Circles Touches Externally



No of common tangents = 3

Q.46 The number of integers greater than 6000 that can be formed, using the digits 3,5,6,7 and 8, without repetition is :

(1) 120 (2) 72 (3) 216 (4) 192

Sol: (4)

$\_\ \_\ \_\ \_\ \_$

$\uparrow$

3 ways

Total $3\times4!+5!=192$

$=72+120=192$

1 st place can be filled in 3 ways with $6(\text{or})7(\text{or})8$ Remaining 4 places in 4! ways and 5 digits can be arranged in 5! ways .

Q.47 Let $y(x)$ be the solution of the differential equation

$(x\log x)\dfrac{dy}{dx}+y=2x\log x,(x\geq1)$. The $y(e)$ is equal to

(1) 2 (2) 2e (3) e (4) 0

Sol: (1)

Given $\frac{dy}{dx}+\left(\frac{1}{2\log x}\right)y=2$

I.F. $=e^{\int\frac{1}{x\log x}dx}$

$=e^{\ell n(\ell n x)}$

$=\ell n\,x$

$\therefore$ soln.is

$y(\ell n\,x)=\int 2\ \ell n\,x\ dx+C$

$y(\ell n\,x)=2x(\ell n\,x-1)+C$ ...(1)

Given $x\ge 1$

At $x=1$

$y(0)=-2+C$

$\Rightarrow C=2$

Sol in (1)

$y(\ell n\ x)=2x(\ell nx-1)+2$

$\therefore$ *put* $x=e,$

$y=2e(0)+2$

$\boxed{y=2}$

Q.48 If $A=\begin{bmatrix}1 & 2 & 2\\ 2 & 1 & -2\\ a & 2 & b\end{bmatrix}$ is a matix satisfying the equation $\mathrm{AA^T}=9\,\mathrm{I}$, where I is 3×3 identity matrix, then the ordered pair (a,b) is equal to :

(1) $(2,1)$ (2) $(-2,-1)$ (3) $(2,-1)$ (4) $(-2,1)$

Sol: (2)

$\mathrm{AA^T}=9\mathrm{I}$

$$\begin{bmatrix}1 & 2 & 2\\ 2 & 1 & -2\\ a & 2 & b\end{bmatrix}\begin{bmatrix}1 & 2 & a\\ 2 & 1 & 2\\ 2 & -2 & b\end{bmatrix}=\begin{bmatrix}9 & 0 & 0\\ 0 & 9 & 0\\ 0 & 0 & 9\end{bmatrix}$$

$$\begin{bmatrix}9 & 0 & a+4+2b\\ 0 & a & 2a+2-2b\\ a+4+2b & 2a+2-2b & a^2+4+b^2\end{bmatrix}=\begin{bmatrix}9 & 0 & 0\\ 0 & 0 & 9\\ 0 & 0 & 9\end{bmatrix}$$

$$\Rightarrow\begin{cases}a+4+2b=0\\ 2a+2-2b=9\end{cases}\Rightarrow(a,b)=(-2,-1)$$

Q.49 If m is the A.M. of two distinct real numbers l and n $(l, n > 1)$ and G_1, G_2 and G_3 are three geometric means between l and n, then $G_1^4 + 2G_2^4 + G_3^4$ equals.

(1) $4lmn^2$ (2) $4l^2m^2n^2$ (3) $4l^2mn$ (4) $4lm^2n$

Sol: (4)

$$m = \frac{l+n}{2}$$

$L, G_1\ G_2 G_3\ n$ are in G.P.

$$n = l(r)^4$$

$$\left(\frac{n}{l}\right)^{1/4} = r$$

$$G_1 = l\left(\frac{n}{l}\right)^{1/4}$$

$$G_2 = l\left(\frac{n}{l}\right)^{2/4}$$

$$G_3 = l\left(\frac{n}{l}\right)^{3/4}$$

$$G_1^4 + 2G_2^4 + G_3^4$$

$$= l^4\left[\frac{n}{l} + 2\left(\frac{n}{l}\right)^2 + \left(\frac{n}{l}\right)^3\right]$$

$$= l^4\left[\frac{nl^2 + 2n^2l + n^3}{l^3}\right]$$

$$= l\left[n\left(l^2 + 2nl + n^2\right)\right]$$

$$= \ln(l+n)^2 = 4m^2 \ln$$

Q.50 The negation of $\sim s \vee (\sim r \wedge s)$ is equivalent to :

(1) $s \vee (r \vee \sim s)$ (2) $s \wedge r$ (3) $s \wedge \sim r$ (4) $s \wedge (r \wedge \sim s)$

Sol: (2)

$$\sim[\sim s \vee (\sim r \wedge s)]$$

$$= s \wedge \sim(\sim r \wedge s)$$

$$= s \wedge (r \vee \sim s)$$

$$= (s \wedge r) \vee (s \wedge \sim s)$$

$$= (s \wedge r) \vee \phi$$

$$= (s \wedge r)$$

Q.51 The integral $\int \frac{dx}{x^2\left(x^4+1\right)^{3/4}}$ equals :

(1) $-\left(x^4+1\right)^{\frac{1}{4}}+c$ (2) $-\left(\frac{x^4+1}{x^4}\right)^{\frac{1}{4}}+c$ (3) $\left(\frac{x^4+1}{x^4}\right)^{\frac{1}{4}}+c$ (4) $\left(x^4+1\right)^{\frac{1}{4}}+c$

Sol: (2)

$$I=\int \frac{dx}{x^5\left(1+\frac{1}{x^4}\right)^{3/4}}$$

Let $1+\frac{1}{x^4}=t^4$

$$\frac{-4}{x^5}dx=4t^3dt$$

$$I=\int -\frac{t^3dt}{t^3}=-t+c$$

$$=-\left(\frac{x^4+1}{x^4}\right)^{1/4}+c$$

Q.52 The normal to the curve $x^2+2xy-3y^2=0$, at $(1, 1)$:

(1) meets the curve again in the third quadrant.

(2) meets the curve again in the fourth quadrant.

(3) does not meet the curve again.

(4) meets the curve again in the second quadrant.

Sol: (2)

$$\frac{dy}{dx}=1$$

$$-\frac{dx}{dy}(x,y)=-1$$

equation of normal at $(1,1)$ is

$$(y-1)=-(x-1)\Rightarrow x+y=2$$

Point of intersection of normal with the curve

$$x^2+2xy-3y^2=0 \qquad ...(1)$$

$$x+y=2 \qquad ...(2)$$

Elliminate y in both the equations

$x=1,3$ hence, $y=1,\ -1$

$(1,1)$ & $(3,-1)$

Hence, meets the curve again in the fourth quadrant.

Q.53 Let $\tan^{-1} y = \tan^{-1} x + \tan^{-1}\left(\frac{2x}{1-x^2}\right)$, where $|x| < \frac{1}{\sqrt{3}}$. Then a value of y is :

(1) $\frac{3x-x^3}{1+3x^2}$ (2) $\frac{3x+x^3}{1+3x^2}$ (3) $\frac{3x-x^3}{1-3x^2}$ (4) $\frac{3x+x^3}{1-3x^2}$

Sol: (3)

$$\tan^{-1} y = \tan^{-1} x + 2\tan^{-1} x$$

$$= 3\tan^{-1} x$$

$$\tan^{-1} y = \tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right)$$

$$y = \frac{3x-x^3}{1-3x^2}$$

Q.54 If the function.

$g(x) = \begin{cases} k\sqrt{x+1}, & 0 \le x \le 3 \\ mx+2, & 3 < x \le 5 \end{cases}$ is differentiable, then the value of $k+m$ is :

(1) $\frac{10}{3}$ (2) 4 (3) 2 (4) $\frac{16}{5}$

Sol: (3)

$\because$ g(x) is differentiable so g(x) must be continuos as well.

L.H.L of g(x) at $x = 3$ is 2K

R.H.L. of g(x) at $x = 3$ is $3m+2$

$\because$ g(x) is continuous at $x = 3$

$$3m + 2 = 2k$$

$$3m - 2k = -2 \qquad ...(1)$$

$$g'(x) = \begin{cases} \frac{k}{2\sqrt{x+1}} & 0 \le x \le 3 \\ m & 3 < x \le 5 \end{cases}$$

L.H.D at $x = 3$ $= \frac{k}{4}$

R.H.D at $x = 3 = m$

$\because$ L.H.D = R. H.D

$$\frac{k}{4} = m$$

$$\boxed{k = 4m} \qquad ....(2)$$

By solving both the equations

$$m = \frac{2}{5} \quad k = \frac{8}{5}$$

Hence $k + m = \frac{2}{5} + \frac{8}{5} = \frac{10}{5} = 2$

Q.55 The mean of the data set comprising of 16 observations is 16. If one of the observation valued 16 is deleted and three new observations valued 3,4 and 5 are added to the data, then the mean of the resultant data, is:

(1) 15.8 (2) 14.0 (3) 16.8 (4) 16.0

Sol: (2)

Let $x_1, x_2, x_3, \ldots\ldots\ldots x_{15}, x_{16}$: variables

$$\frac{x_1 + x_2 + \ldots\ldots\ldots\ldots + x_{16}}{16} = 16$$

$x_1 + \ldots\ldots\ldots\ldots + x_{16} = 16.16$

As per new conditions, Let $x_{16} = 16$

$$\frac{(x_1 + x_2 + \ldots\ldots\ldots + x_{15}) + (2+3+4)}{18} = \frac{(16.16 - 16) + 2 + 3 + 4}{18}$$

$$\overline{X} = \frac{252}{18} = 14.0$$

Q.56 The integral $\int_2^4 \frac{\log x^2}{\log x^2 + \log(36 - 12x + x^2)} dx$ is equal to :

(1) 1 (2) 6 (3) 2 (4) 4

Sol: (1)

$$I = \int_2^4 \frac{\log x^2}{\log x^2 + \log(6-x)^2} dx \quad \ldots (1)$$

Using property $\int_a^b f(a+b-x)dx = \int_a^b f(x)dx$

$$I = \int_2^4 \frac{\log(6-x)^2}{\log(6-x)^2 + \log x^2} dx \quad \ldots (2)$$

(1) + (2) $\Rightarrow$

$$I = \frac{1}{2}\int_2^4 \frac{\cancel{\log x^2 + \log(6-x)^2}}{\cancel{\log x^2 + \log(6-x)^2}} dx$$

$$I = \frac{1}{2}\int_2^4 dx = \frac{1}{2}[4-2] = 1$$

Q.57 Let α and β be the roots of equation $x^2 - 6x - 2 = 0$. If $a_n = \alpha^n - \beta^n$, for $n \geq 1$, then the value $\frac{a_{10} - 2a_8}{2a_9}$ is equal to :

(1) 3 (2) –3 (3) 6 (4) –6

Sol: (1)

$x^2 - 6x - 2 = 0$ has roots α, β

$\alpha + \beta = 6, \qquad \alpha \cdot \beta = -2 \qquad \ldots (1)$

$a_n = \alpha^n - \beta^n \quad \forall \ x \geq 1$

Now $\frac{a_{10}-2\cdot a_8}{2\cdot a_9}$

$$=\frac{\left(\alpha^{10}-\beta^{10}\right)-2\left(\alpha^8-\beta^8\right)}{2\cdot\left(\alpha^9-\beta^9\right)}$$

Using -2 as $\alpha\cdot\beta$

$$\Rightarrow\frac{\left(\alpha^{10}-\beta^{10}\right)+\alpha\cdot\beta\left(\alpha^8-\beta^a\right)}{2\cdot\left(\alpha^9-\beta^9\right)}$$

$$\Rightarrow\frac{\alpha^9\left(\alpha+\beta\right)-\beta^9\left(\beta+\alpha\right)}{2\cdot\left(\alpha^9-\beta^9\right)}$$

$$\Rightarrow\frac{\left(\alpha^9-\beta^9\right)\cdot\left(\alpha+\beta\right)}{2\cdot\left(\alpha^9-\beta^9\right)}=\frac{\alpha+\beta}{2}\quad\Rightarrow\frac{6}{2}=3$$

Q.58 Let $f\left(x\right)$ be a polynomial of degree four having extreme values at x = 1 and x =2. If $\lim_{x\to0}\left(1+\frac{f\left(x\right)}{x^2}\right)=3$, then $f(2)$ is equal to :

(1) 0 (2) 4 (3) –8 (4) –4

Sol: (1)

$\lim_{x\to0}\left(1+\frac{f\left(x\right)}{x^2}\right)=3$, $f(x)$ has repeated root at 0

$$\Rightarrow f\left(x\right)=x^2.\left(ax^2+bx+c\right)$$

$$\Rightarrow\lim_{x\to0}\left(1+\frac{x^2.\left(ax^2+bx+c\right)}{x^2}\right)=3$$

$$\Rightarrow c+1=3\Rightarrow c=2$$

$f\left(x\right)$ has extreme at $x=1$ and $x=2$

$$\Rightarrow f'\left(x\right)=0\ \text{ at }\ x=1,2$$

$$\Rightarrow f'\left(1\right)=0\Rightarrow 4a+3b+4=0 \qquad ...(1)$$

$$f'\left(2\right)=0\Rightarrow 8a+3b+2=0 \qquad ...(2)$$

(1) and (2) $\Rightarrow a=\frac{1}{2}, b=-2$

$$\Rightarrow f\left(x\right)=x^2\left(\frac{x^2}{2}-2x+2\right)$$

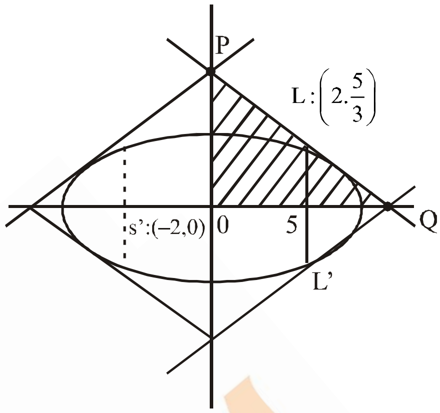
$$f\left(2\right)=4.\left(4-4\right)=0$$

Q.59 The area (in sq. units) of the quadrilateral formed by the tangents at the end points of the latera recta to the ellipse $\frac{x^2}{9}+\frac{y^2}{5}=1$, is :

(1) $\frac{27}{2}$ (2) 27 (3) $\frac{27}{4}$ (4) 18

Sol: (2)

Ellipse $\frac{x^2}{9}+\frac{y^2}{5}=1$



$e=\sqrt{1-\frac{b^2}{a^2}}$

$=\sqrt{1-\frac{5}{9}}$

$=\frac{2}{3}$

focus : $(\pm ae, o)$

$=(\pm 2, 0)$

Latus rectum co-ordinates : $\left(\pm\ ae, \pm\frac{b^2}{a}\right)$

$\Rightarrow\left(\pm 2, \pm\frac{5}{3}\right)$

Tangent at L : $\frac{x\cdot 2}{9}+\frac{y\cdot\frac{5}{3}}{5}=1$

$Q:\left(\frac{9}{2}, 0\right)$

$P:(0,3)$

Area of quadrilateral : $4(\Delta OPQ)$

$=4\cdot\frac{1}{2}\cdot\frac{9}{2}\cdot 3$

$=\ 27$

Q.60 If 12 identical balls are to be placed in 3 identical boxes, then the probability that one of the boxes contains exactly 3 balls is :

(1) $220\left(\frac{1}{3}\right)^{12}$ (2) $22\left(\frac{1}{3}\right)^{11}$ (3) $\frac{55}{3}\left(\frac{2}{3}\right)^{11}$ (4) $55\left(\frac{2}{3}\right)^{10}$

Sol: (3)

Probability is a ratio hence, answer will not be effected when given balls are different.

$n(s) = 3^{12}$ since each ball will go in three ways.

Event : select 3 balls out of 12 in ${}^{12}C_3$ ways.

Remaining 9 balls will be placed in other 2 boxes in 2^9 ways.

$\therefore\ n(E) = {}^{12}C_3 \times 2^9$

$P(E) = n(E) / n(s)$

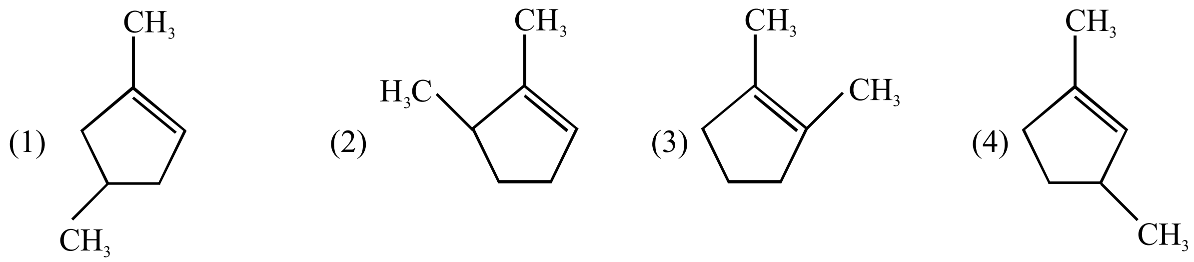
$$= \frac{{}^{12}C_3 \times 2^9}{3^{12}}$$

$$= \frac{220 \times 2^9}{3^{12}} = \frac{55}{3}\left(\frac{2}{3}\right)^{11}$$

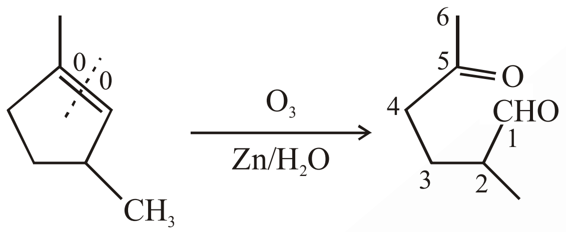
(**Note :** Assumption is atleast 1box contains 3 balls but not exactly)

CHEMISTRY

Q.61 Which compound would give 5-keto-2-methyl hexanal upon ozonolysis ?



Sol. (4)



5-keto-2-methyl hexanal

Q.62 Which of the vitamins given below is water soluble ?

(1) Vitamin E (2) Vitamin K (3) Vitamin C (4) Vitamin D

Sol. (3)

Water soluble vitamins are B and 'C'

Fat soluble – A, D, E, K

Q.63 Which one of the following alkaline earth metal sulphates has its hydration enthalpy greater than its lattice enthalpy ?

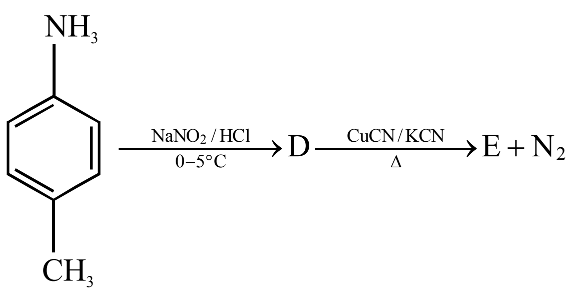
(1) $BaSO_4$ (2) $SrSO_4$ (3) $CaSO_4$ (4) $BeSO_4$

Sol. (4)

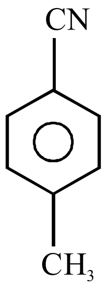
$BeSO_4$

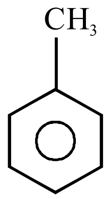
Be^{2+} hydration energy is greater than lattice energy due to its covalent nature and hence more soluble in water.

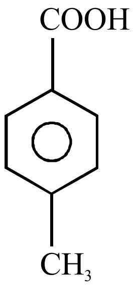
Q.64 In the reaction

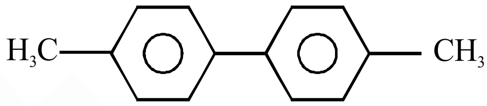


the product E is :

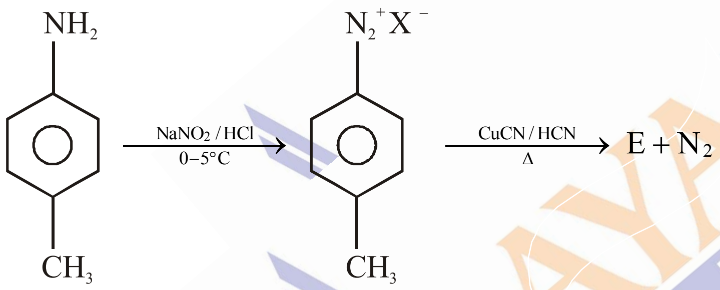
(1) 

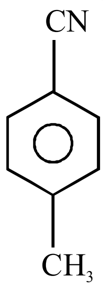
(2) 

(3) 

(4) 

Sol. (1)



E =  (Sandmeyer's Reaction)

Q.65 Sodium metal crystallizes in a body centred cubic lattice with a unit cell edge of 4.29Å. The radius of sodium atom is approximately :

(1) 5.72Å (2) 0.93Å (3) 1.86Å (4) 3.22Å

Sol. (3)

In B.C.C,

$$4r = a\sqrt{3}$$

$$r = \frac{4.29\sqrt{3}}{4}$$

$$= 1.857\text{Å} = 1.86\text{Å}$$

Q.66 Which of the following compounds is not colored yellow ?

(1) $(NH_4)_3 [As(Mo_3O_{10})_4]$ (2) $BaCrO_4$

(3) $Zn_2[Fe(CN)_6]$ (4) $K_3[Co(NO_2)_6$

Sol. (3)

$Zn_2[Fe(CN)_6]$ — Bluish White ppt

$(NH_4)_3 [As(Mo_3O_{10})_4]$ — Yellow ppt

Ammonium Arseno Molybdate

$BaCrO_4$ — Yellow ppt

Q.67 Which of the following is the energy of a possible excited state of hydrogen ?

(1) – 3.4 eV (2) + 6.8 eV (3) + 13.6 eV (4) – 6.8 eV

Sol. (1)

$$E_n = -13.6\frac{Z^2}{n^2}\text{eV}$$

$$= -\frac{13.6\times1^2}{4}\text{eV}$$

$$= -3.4 \text{ eV}$$

Q.68 Which of the following compounds is not an antacid ?

(1) Phenelzine (2) Ranitidine

(3) Aluminium hydroxide (4) Cimetidine

Sol. (1)

Phenelzine — Tranquilizer (not an antacid)

Ranitidine, $Al(OH)_3$, Cimetidine are antacids

Q.69 The ionic radii (in Å) of N^{3-}, O^{2-} and F^- are respectively :

(1) 1.71, 1.40 and 1.36 (2) 1.71, 1.36 and 1.40

(3) 1.36, 1.40 and 1.71 (4) 1.36, 1.71 and 1.40

Sol. (1)

Ionic radii (Å) of N^{3-}, O^{2-} and F^- are 1.71Å, 1.40Å and 1.36Å

Ionic radii α – ve charge of anion.

Q.70 In the context of the Hall-Heroult process for the extraction of Al, which of the following statements is false?

(1) Al^{3+} is reduced at the cathode to form Al

(2) Na_3AlF_6 serves as the electrolyte

(3) CO and CO_2 are produced in this process

(4) Al_2O_3 is mixed with CaF_2 which lowers the melting point of the mixture and brings conductivity

Sol. (4)

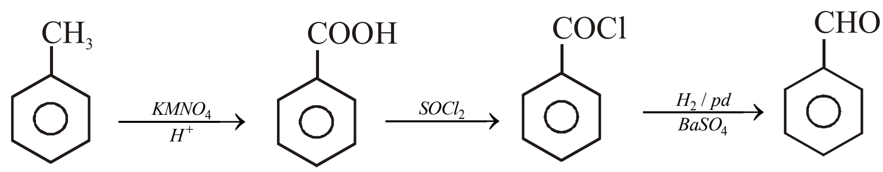
Conceptual.

Q.71 In the following sequence of reactions :

Toluene $\xrightarrow{KMnO_4} A \xrightarrow{SOCl_2} B \xrightarrow[BaSO_4]{H_2/Pd} C$, the product C is:

(1) $C_6H_5CH_2OH$ (2) C_6H_5CHO (3) C_6H_5COOH (4) $C_6H_5CH_3$

Sol: (2)



(Rosenmund's reduction)

Q.72 Higher order (>3) reactions are rare due to :

(1) shifting of equilibrium towards reactants due to elastic collisions
(2) loss of active species on collision
(3) low probability of simultaneous collision of all the reacting species
(4) increase in entropy and activation energy as more molecules are involved

Sol: (3)

Higher order (>3) reactions are rare due to low probability of simultaneous collision of an reacting species.

Q.73 Which of the following compounds will exhibit geometrical isomerism?

(1) 2-Phenyl-1-butene (2) 1,1-Diphenyl-1-propane
(3) 1-Phenyl-2-butene (4) 3-Phenyl-1-butene

Sol: (3)

$$CH_2 = \overset{\overset{\displaystyle Ph}{|}}{C} - CH_2 - CH_3$$ – not possible

$$Ph - CH_2 - CH = CH - CH_3$$

Only exhibits geometrical isomrism.

Q.74 Match the catalysts to the correct processes:

Catalyst	**Process**
(A) $TiCl_3$	(i) Wacker process
(B) $PdCl_2$	(ii) Ziegler - Natta polymerization
(C) $CuCl_2$	(iii) Contact process
(D) V_2O_5	(iv) Deacon's process

(1) (A)-(ii), (B)-(iii), (C)-(iv), (D) -(i) (2) (A)-(iii), (B)-(i), (C)-(ii), (D) -(iv)
(3) (A)-(iii), (B)-(ii), (C)-(iv), (D) -(i) (4) (A)-(ii), (B)-(i), (C)-(iv), (D) -(iii)

Sol: (4)

(A) $TiCl_3$	(ii) Ziegler - Natta polymerization
(B) $PdCl_2$	(i) Wacker process
(C) $CuCl_2$	(iv) Deacon's process
(D) V_2O_5	(iii) Contact process

Q.75 The intermolecular interaction that is dependent on the inverse cube of distance between the molecules is:

(1) London force (2) hydrogen bond

(3) ion- ion interaction (4) ion-dipole interaction

Sol: (2)

Dipole - Dipole interaction energy between stationary, polar molecules is proportional to

$\frac{1}{r^3}, r =$ distance between molecules.

Q76. The molecular formula of a commercial resin used for exchanging ions in water softening is $C_8H_7SO_3Na$ (Mol. wt. 206). Ca^{2+} ions by the resin when expressed in mole per gram resin ?

(1) $\frac{2}{309}$ (2) $\frac{1}{412}$ (3) $\frac{1}{103}$ (4) $\frac{1}{206}$

Sol: (2)

$$\underset{\substack{(206\,g)\\2\,mol}}{2C_8H_7SO_3Na} + \underset{1\,mol}{Ca^{2+}} \longrightarrow$$

$1\,mol\ Ca^{2+}$ – 2 mol resin

– 412 g resin

Maximum Ca^{2+} uptake $= \frac{1\,mol}{412} / g$ of resin.

Q.77 Two Faraday of electricity is passed through a solution of $CuSO_4$. The mass of copper deposited at the cathode is :

(at. mass of Cu = 63.5 amu)

(1) 2 g (2) 127 g (3) 0 g (4) 63.5 g

Sol: (4)

$1F \longrightarrow 1\,eq.\ wt.$

$$Cu^{2+} + \underset{2F}{2e^-} \longrightarrow \underset{1\,mol}{Cu}$$

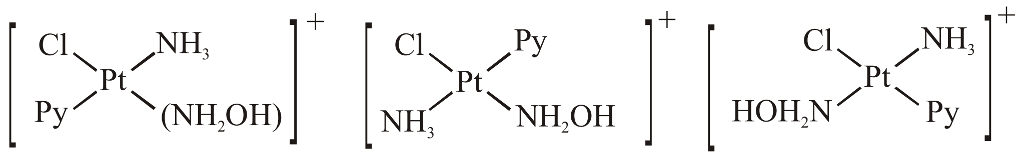
2 Faraday's electricity deposit $\rightarrow$ 63.5 g of Cu.

Q.78 The number of geometric isomers that can exist for square planar $\left[Pt(Cl)(py)(NH_3)(NH_2OH)\right]^+$ is $(py = \text{pyridine})$:

(1) 4 (2) 6 (3) 2 (4) 3

Sol: (4)

The no. of geometrical Isomers for square planar are 3



Q.79 In Carius method of estimation of halogens, 250 mg of an organic compound gave 141 mg of AgBr. The percentage of bromine in the compound is:
(at. mass Ag = 108; Br = 80)

(1) 48 (2) 60 (3) 24 (4) 36

Sol: (3)

Molar mass of $AgBr = 188g$

$188\,g\ AgBr = 80g\ Br_2$

$141mg\ AgBr = \frac{80}{188}\times 141\ mg\ Br_2$

$= 60\,mg\ Br_2$

$\%\ Br = \frac{60}{250}\times 100 = 24\%$

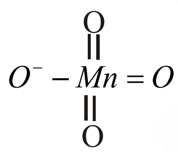
or

$\%\ Br = \frac{80}{188}\times\frac{141}{250}\times 100 = 24\%$

Q.80 The colour of $KMnO_4$ is due to:

(1) $L \to M$ charge transfer transition (B) $\sigma \to \sigma^*$ transition
(3) $M \to L$ charge transfer transition (4) $d-d$ transition

Ans: (1)



colour due to

$L \to M$ charge transfer transition.

Q.81 The synthesis of alkyl fluorides is best accomplished by :

(1) Finkelstein reaction (2) Swarts reaction
(3) Free radical fluorination (4) Sandmeyer's reaction

Sol: (2)

$R-Br + AgF \to AgBr\downarrow + R-F$

Swart's reaction

Q.82 3g of activated charcoal was added to 50mL acetic solution (0.06N) in a flask. After an hour it was filtered and the strength of the filtrate was found to be 0.042 N. The amount of acetic acid adsorbed (per gram charcoal) is :

(1) 42 mg (2) 54 mg (3) 18 mg (4) 36 mg

Sol: (3)

No. of meq. of CH_3COOH given $= (0.06\times 50)$ m eq.

No. of meq. of CH_3COOH remaining $= (0.06\times 50)$ m eq.

meq of CH_3COOH absorbed $= (0.06\times 50) - (0.042\times 50)$

$= \frac{50}{1000}\times 0.018\ eq$

$$= \frac{0.900}{1000} = 9 \times 10^{-4} eq$$

$W_{CH_3COOH} = 9 \times 10^{-4} \times 60 = 54mg$ for 3 g of charcoal

$\therefore CH_3COOH$ adsorbed by 1 g of charcoal $= \frac{54}{3} = 18\,g$

Q.83 The vapour pressure of acetone at 20^0C is 185 torr. When 1.2 g of a non – volatile substance was dissolved in 100 g of acetone at 20^0C, its vapour pressure was 183 torr. The molar mass (g mol^{-1}) of the substance is

(1) 128 (2) 488 (3) 32 (4) 64

Sol: (4)

vapour pressure of Acetone = 185 torr

molar mass of Acetone(CH_3COCH_3) = 58

$$n_{acetone} = \frac{100}{58} = 1.724$$

$$n_{substance} = \frac{1.2}{M}$$

by $\frac{P^0 - P_s}{P^0} = \frac{n_{substance}}{n_{substance} + n_{acetone}}$

$$\frac{185-183}{185} = \frac{\frac{1.2}{M}}{\frac{1.2}{M} + 1.724}$$

$$\frac{2}{185} = \frac{\frac{1.2}{M}}{\frac{1.2}{M} + 1.724}$$

$$\frac{185}{2} = 1 + \frac{1.724}{1.2/M}$$

$$\frac{185-2}{2} = \frac{1.724}{1.2} M$$

$$M = 63.68 \approx 64$$

Q.84 Which among the following is the most reactive ?

(1) I_2 (2) ICl (3) Cl_2 (4) Br_2

Sol: (2)

Most reactive are inter halogen compounds than halogens (except F_2) due to weak bond dissociation energy and polarity.

$\therefore ICl$

Q.85 The standard Gibbs energy change at 300K for the reaction $2A \rightleftharpoons B+C$ is 2494.2 J. At a given time, the composition of the reaction mixture is $[A]=\frac{1}{2},[B]=2$ and $[C]=\frac{1}{2}$. The reaction proceeds in the :

$[R=8.314\,J/K/mol, e=2.718]$

(1) forward direction because $Q<K_C$

(2) reverse direction because $Q<K_C$

(3) forward direction because $Q>K_C$

(4) reverse direction beacuse $Q>K_C$

Sol: (4)

$$Q=\frac{[B][C]}{[A]^2}=\frac{2\times\frac{1}{2}}{\frac{1}{2}\times\frac{1}{2}}=4$$

$$\Delta G^0=-2.303RT\log K_{eq}$$

$$2494.2=-2.303\times 8.314\times 300\times \log K$$

$$-\log K=0.434$$

$$\log\frac{1}{K}=0.434$$

$$\frac{1}{K}=2.7$$

$$K=\frac{1}{2.7}=0.37$$

$\therefore Q>K$

$\therefore$ Backward reaction favoured

i.e. reverse direction

Q.86 **Assertion :** Nitrogen and oxygen are the main components in the atmosphere but these do not react to form oxides of nitrogen.

Reason : The reaction between nitrogen and oxygen requires high temperature.

(1) The assertion is incorrect, but the reason is correct

(2) Both the assertion and reason are incorrect

(3) Both assertion and reason are correct, and the reason is the correct explanation for the assertion

(4) Both assertion and reason are correct, but reason is not the correct explanation for the assertion

Sol: (3)

Assertion and Reason are correct & the reason is the correct explanation for the assertion.

$N_2+O_2 \rightleftharpoons 2NO$

(occurs only during lightening)

Q.87 Which one has the highest boiling point ?

(1) Kr (2) Xe (3) He (4) Ne

Sol: (2)

As the surface area & molecular weight increases, the Vanderwall's forces increases & B.P. increases.

$\therefore$ Xe.

Q.88 Which polymer is used in the manufacture of paints and lacquers ?

(1) Polypropene (2) Poly vinyl chloride (3) Bakelite (4) Glyptal

Sol: (4)

Glyptal is used in paints.

Q.89 The following reaction is performed at 298K. $2NO(g)+O_2(g) \rightleftharpoons 2NO_2(g)$

The standard free energy of formation of NO(g) is 86.6 kJ/mol at 298 K. What is the standard free energy of formation of NO_2(g) at 298K ? $\left(K_p = 1.6\times10^{12}\right)$

(1) $86600-\dfrac{\ln\left(1.6\times10^{12}\right)}{R(298)}$ (2) $0.5\left[2\times86,600-R(298)\ln\left(1.6\times10^{12}\right)\right]$

(3) $R(298)\ln\left(1.6\times10^{12}\right)-86600$ (4) $86600+R(298)\ln\left(1.6\times10^{12}\right)$

Sol: (2)

$$\Delta G_R = -RT\ln Kp$$

$$\Delta G_R = 2\Delta_f G_{NO_2} - 2\Delta G^0{}_{NO}$$

$$-RT\ln 1.6\times10^{12} = 2\left(\Delta_f G_{NO_2} - 86,600\right)$$

$$86,600-\frac{RT\ln\left(1.6\times10^{12}\right)}{2} = \Delta_f G_{NO_2}$$

$$\Delta_f G_{NO_2} = 0.5\left[2\times86600 - R(298)\ln\left(1.6\times10^{12}\right)\right]$$

Q.90 From the following statements regarding H_2O_2, choose the incorrect statement :

(1) It has to be stored in plastic or wax lined glass bottles in dark

(2) It has to be kept away from dust

(3) It can act only as an oxidizing agent

(4) It decomposes on exposure to light

Sol: (3)

It can acts not only as oxidising agent but also as reducing agent.